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Title: An Overview of the Flinders Hamiltonian Cycle Project
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We consider the famous Hamiltonian cycle problem (HCP) from the perspective of approaches studied by members of the Flinders Hamiltonian Cycle Project. In particular, we shall briefly outline results from three, somewhat overlapping, lines of investigations.
The first of these stems from an embedding of the HCP in a Markov decision process (MDP). More specifically, we consider the HCP as an optimization problem over the space of occupational measures induced by the MDP's stationary policies. The shape and volume of that space will depend on a small parameter that induces a number of interesting phenomena as it approaches zero.
The second line of research describes the development of a very successful "Snakes-and-Ladders Heuristic" (SLH) for finding a Hamiltonian cycle in an undirected graph. Despite the fact that HCP is known to be NP-complete, this polynomial complexity heuristic has proved itself to be extremely reliable in solving Hamiltonian graphs even in cases where the number of Hamiltonian cycles is extremely small relative to the number of potential candidate solutions.
The third line, aims to correctly identify non-Hamiltonian cubic graphs via polynomial complexity heuristics. The latter leads to a suitably constructed parametrized polytope of discounted occupational measures that can be used as a domain where Hamiltonian solutions are sought. It is known that whenever a given graph possesses Hamiltonian cycles, these correspond to certain extreme points of that polytope. In addition, we consider the limiting - parameter free - polyhedral region that can serve as a basis for determining non-Hamiltonicity of cubic graphs. Numerical results indicate that determining non-Hamiltonicity of a great majority of non-Hamiltonian cubic graphs is likely to be a problem of polynomial complexity despite the fact that HCP is known to be NP-complete already for cubic graphs.