Abstract submitted for Universal Structures in Mathematics and Computing

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Title: Heyting algebras with operators
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Session: abstract

It is well-known that congruences on a Heyting algebra are determined by filters on the underlying lattice. If an algebra $\mathbf{A}$ has a Heyting algebra reduct, it is of natural interest to characterise the filters that correspond to congruences on $\mathbf{A}$. Such a characterisation was given by Hasimoto, calling them *normal filters*. When normal filters can be described using a single unary term, many useful properties come to life. In general, a unary term that determines normal filters will be called a *normal filter term*. The traditional example comes from boolean algebras with operators (BAOs).
A algebra $\mathbf{B} = \langle B; \vee, \wedge, \neg, \{f_i \mid i \in I\}, 0, 1\rangle$ is a *boolean algebra with (dual) operators* if $\langle B; \vee, \wedge, \neg, 0, 1\rangle$ is a boolean algebra, and for each $i \in I$, the operation f_i is a unary normal operator, i.e., f_i is a map satisfying $f_i 1 = 1$ and $f_i(x \wedge y) = f_i x \wedge f_i y$. Conventionally, a BAO is defined dually, but it turns out that meet-preserving operations are more natural for Heyting algebras. If $\mathbf{B}$ is of finite type, then congruences on $\mathbf{B}$ are determined by filters closed under the map d, defined by

$$dx = \bigwedge\{f_i x \mid i \in I\}.$$

This is easily generalised to the case that each f_i is of any finite arity. Hasimoto gave a construction which generalises the term above to Heyting algebras equipped with an arbitrary set of operations. The construction does not apply in all cases, and even when it does, it does not necessarily produce a term function on the algebra. Having said that, Hasimoto proved that his construction guarantees a normal filter term for Heyting algebras with operators.
In this talk, we will extend Hasimoto's constraints to provide normal filter terms for a wider class of algebras. We will also speak about double-Heyting algebras, for which it is not known if Hasimoto's construction applies. Despite this, they are known to possess a normal filter term by a result of Sankappanavar. Finally, we will see how this can be used to prove that, for

dually pseudocomplemented Heyting algebras, a variety $\mathcal{V}$ is semisimple if and only if $\mathcal{V}$ is a discriminator variety.