Abstract submitted for Universal Structures in Mathematics and Computing

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Title: Constellations: Arrows Without Targets.
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Session: abstract

Constellations are partial algebras that are one-sided generalisations of categories. Categories model classes of objects together with suitably defined mappings between them. Each mapping, or arrow, has a domain and codomain (source and target), and composition of mappings $f \cdot g$ is defined precisely when the codomain of f coincides with the domain of g. An alternative notion of composition arises if one only requires the codomain of f to be a *subset* of the domain of g. When this is done, precise information about codomains is no longer needed, and "arrows" have sources but no targets. This is more natural in many examples, for example all mappings between sets having infinite domain but arbitrary image. The abstract concept corresponding to these concrete examples is that of a constellation, a concept first introduced by Gould and Hollings (who showed that the category of so-called inductive constellations is isomorphic to the category of left restriction semigroups).

Here we consider constellations in full generality, giving many examples. We characterise those small constellations that are isomorphic to constellations of partial functions, as well as those constellations that arise as (sub-)reducts of categories, and show that categories are nothing but two-sided constellations. We demonstrate that the naive notion of substructure can be captured within constellations but not within categories. We show that every constellation P gives rise to a category $\mathcal{C}(P)$, its "canonical extension", in a simplest possible way, that P is a quotient of $\mathcal{C}(P)$ obtained by factoring out a so-called canonical congruence, and that many familiar concrete categories may be constructed from simpler quotient constellations in this way. A correspondence between constellations and categories equipped with a canonical congruence is established.

This is joint work with Victoria Gould.