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Title: Enumerating (Garside) lattices
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Generating catalogues of examples that are in some sense complete has proved to be an important step towards understanding algebraic concepts, and it often is one of the key steps towards a successful classification and thus a complete theory.
One type of algebraic structure I am particularly interested in are so-called *Garside monoids*: This is a class of infinite monoids that admit a "nice" (and effectively computable) normal form for their elements. The notion of Garside monoids captures and unifies many important examples, for instance all Artin groups of spherical type, free groups, free abelian groups, as well as (at least some) mapping class groups, complex reflection groups, and affine Artin groups.

Garside monoids can be defined by a finite lattice (in the meaning of combinatorics), together with a labelling of the edges that satisfies certain conditions. Thus, one can catalogue the Garside monoids up to some size threshold by:

(a) Constructing all lattices on at most n points.

(b) Constructing all suitable edge labellings of a given finite lattice.

It turns out that (b) isn't too bad. (In the unlikely case that there is time, I'll mention the key ideas.)
Unfortunately, (a) is: even the *number* of lattices on n points is currently only known for $n \leq 19$ (OEIS: A006966).

In my talk, I'll focus on explaining how to combine group theory, combinatorics and some clever ideas from computing to enumerate unlabelled lattices more effectively.